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A MODEL STUDY OF INTERNAL WAVES

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A MODEL STUDY OF INTERNAL WAVES

by

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A thesis submitted in partial fulfillment

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ABSTRACT

Experiments on the generation of internal waves by low amplitude surface waves traveling over a continental shelf were carried out in a U-shaped wave channel 10.4 m long, 0.4 m wide and up to 0.4 m deep. Long surface waves were generated at one end of this channel, and various model continental shelves were placed at the other end. The shelves were either of uniform depth or of constant slope, and their height above the bottom and width could be varied over wide ranges. An approximate two-layer system was achieved by means of a fresh water layer introduced over a saline layer. The salinity profile was determined by titration of microsamples drawn from a series of depths. Amplitude and period of surface waves, depth, width, and shape of the shelf, density distribution and total depth were varied independently. Quantitative data on the waves were obtained from both moving and still photographs of dye on the interface between the two layers. The experiments have demonstrated the effectiveness of this mechanism in generating large amplitude internal waves. Friction was important in many of the cases studied, and often caused the internal waves over the shelf to be progressive waves traveling toward the shore, instead of the standing waves predicted by theory. Progressive waves were always found to travel seaward from the continental slope region. The amplitude of the internal waves was found to be in general agreement with the theory of Rattray (1960), when friction was not important in the model. In many cases active mixing took place in the shelf region, and in some cases density currents were formed.

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CHAPTER 1. INTRODUCTION

1.1 Purpose of the Experiment

The purpose of this experiment was to study the internal waves generated by low amplitude surface waves passing over a continental shelf break in a model. An attempt was made to verify Rattray's (1960) theory of internal wave generation by modeling, as nearly as possible, the conditions that were assumed in the theory.

1.2 Importance of Internal Waves

Internal waves have been a subject of renewed interest in the last ten years, and several new theories have been developed concerning their generating mechanism and occurrence in the ocean.

Internal waves in the oceans are probably not isolated phenomena, but represent the normal state of affairs where any density stratification is present (Fjeldstad, 1935). The wide distribution of internal waves is partly explained by the small amount of energy that is required to generate them, which in turn is illustrated by the small amount of energy they contain. For a two-layer system where the difference in density is $\Delta\rho$ and ρ is the density of the lower layer, the ratio of energies of an internal wave to a surface wave of the same amplitude is $\frac{\Delta\rho}{\rho}$. In the model studied this ratio was 0.02, while in the ocean, where the difference in densities is smaller, the value would be nearer 0.002.

Seiwell (1939) has demonstrated that errors of up to 100 percent of the current speed can be caused by internal waves when currents are calculated from dynamic heights computed from data collected at only one time. The depth of a particular density surface may be in error by an

amount equal to the amplitude of the internal wave, and these waves may have an amplitude of as much as 200 meters. Mortimer (1952) has done extensive work in fresh water lakes of such temperature stratification that conditions were favorable for the development of internal waves. His work points out that the assumption of stationary conditions can lead to false conclusions on the distribution of organisms and physical variables.

1.3 Brief History of Internal Wave Theories and Observations

The basic theory of internal waves in a two-layer system was developed over a hundred years ago by Stokes (1847), but over fifty years passed before internal waves were observed in the ocean (Helland-Hansen and Nansen, 1909).

Sandstrom (1908) proposed that changes in wind speed and in atmospheric pressure can cause internal waves in a stratified ocean, and Haurwitz (1953) has advanced a theory of internal wave generation based on the shifting of surface wind direction. In his studies of lakes, Mortimer (1952) found that variations in the wind speed and direction were the most likely cause of internal waves. Other causes of internal waves which have been postulated include the velocity shear between two superposed layers of water (Haurwitz, 1948), seismic activity (Haurwitz, 1953), and bottom friction acting on the lower layer of water as it flows over a changing bottom (Ekman, 1931). Numerous observations of internal waves of short period (less than 4 hours) have been made, but no satisfactory theory for their generating mechanism has been developed.

The above mechanisms are not periodic, but many observations of internal waves in the ocean show them to have approximate periods of 12 or 24 hours. When the inertial period for the particular location is somewhat

different from the tidal period, it is reasonable to assume that tidal influences may be a major cause of these internal waves.

There has been considerable controversy over the importance of internal waves of semidiurnal or diurnal period in the open ocean. Ekman (1931) shows very clearly how it is possible for harmonic analysis to yield internal wave periods of semidiurnal or diurnal period when actually there is no real periodicity to the disturbances. This result may arise from a lack of sufficiently frequent measurements and from the prejudice of the investigator. Haurwitz (1952, 1953) discussed the problems of analyzing internal wave records to determine the significant components, but both Haurwitz and Ekman admit the existence of a few valid observations and analyses of internal waves of tidal period in the open sea and of many more in restricted areas or near coasts.

Early investigators believed that it was possible for tide generating forces to generate internal waves directly. More recently, Defant (1950) and Haurwitz (1950) show that under certain circumstances the rotation of the earth establishes a resonance condition which might permit direct generation of internal waves of greater amplitude than the surface tidal waves. Other investigators including Ekman (1931), and more recently Fjeldstad (1952), have taken the opposite viewpoint, that tide generating forces do not generate internal waves, since these forces act similarly on the whole water column.

It is possible that tide generating forces generate internal waves as a secondary phenomenon. Zeilon (1912) developed a theory of internal wave generation by the flow of a tidal current over a bottom ridge. His theory was an important advance in the study of internal waves, but it is inadequate to explain the presence of internal waves in many areas where

there are no large bottom ridges. In a later paper Zeilon (1934) suggested that tidal currents passing over a continental shelf would also generate internal waves, but he did not develop this idea into a quantitative theory. Since continental shelves are a more common feature of the oceans than large ridges, a theory utilizing shelves might offer a better explanation of the wide occurrence of internal waves. Rattray (1960) developed a quantitative theory for the generation of internal waves at a continental shelf break which will be discussed in later sections of this paper.

Support for the theory of generation of internal waves at the continental shelf break is given by a survey made near the Southern California coast (Emery and Summers, 1960). Observations of the level of isothermal surfaces were made at six stations at frequent intervals over a two-day span. Internal waves of tidal period were observed at three of the stations, but the amplitude and phase were not the same. It appears that variations in bottom topography and variations in the distribution of variables can account for the different characteristics (or for the absence) of the internal waves.

If the internal waves of tidal period are generated at the shelf break it seems likely that they would be dissipated by the time they reached the central parts of the ocean. In some cases it appears that they are in fact dissipated, since Reid (1956) found internal waves 40 miles off the California coast but could not find them 120 miles off the coast. Rudnick and Cochrane (1951) found only slight indications of internal waves in the Pacific Ocean at stations about 150 miles from the coast. Rattray (1957) has suggested that under certain conditions internal waves may travel distances comparable to the width of most ocean basins with only a 50 percent decrease in amplitude. The decrease in amplitude would depend on the

distribution of oceanographic variables, so there is no reason to expect that internal waves generated at the shelf could always be traced long distances seaward of the coast.

Most of the difficulty in determining the importance and distribution of internal waves in the open ocean results from the lack of sufficient data. It is difficult and expensive to measure internal waves because a ship must remain at one station for several days or a buoy mounted instrument must be used. Instruments have been developed that make it much easier to gather internal wave data, although there is still much room for improvement. These instruments include bathythermographs, thermistors, and a more complicated device named the isotherm follower (LaFond, 1961).

1.4 Rattray's Theory of Internal Wave Generation

1.4.1 Assumptions of the Theory

To understand the results of this experiment it is necessary to give some elements of the theory on the generation of internal waves as developed by Rattray (1960). The theory states that in a two-layer system low amplitude surface waves in passing over a discontinuity in depth will generate internal waves.

For mathematical simplicity the following conditions are assumed: (1) the ocean is a two-layer system with the difference in density between the layers small, (2) depth contours are essentially parallel to the crests of the surface tide or wave, (3) the presence of a continental slope causes a rapid change in depth, (4) friction and vertical acceleration are negligible, and (5) no properties vary in a direction parallel to the wave crests. The additional assumption that the effect of the Coriolis parameter is negligible is possible when the theory is applied to the model.

1.4.2 Development of the Equations

With the above assumptions the equation of motion for the upper layer is

$$\frac{\partial u}{\partial t} = -g\frac{\partial \zeta}{\partial x} ,$$

and for the lower layer

$$\frac{\partial u'}{\partial t} = -g\left(1-\frac{\Delta\rho}{\rho}\right)\frac{\partial \zeta}{\partial x} - g\frac{\Delta\rho}{\rho}\frac{\partial \zeta'}{\partial x} .$$

The equation of continuity for the upper layer is

$$\frac{\partial}{\partial x}(hu) = \frac{\partial \zeta'}{\partial t} - \frac{\partial \zeta}{\partial t} ,$$

and for the lower layer

$$\frac{\partial}{\partial x}(h'u') = -\frac{\partial \zeta'}{\partial t} .$$

In the above equations u is the horizontal velocity, h is the depth of the upper layer, h' is the depth (variable) of the lower layer, l is the shelf width, D is the depth off the shelf, d is the depth over the shelf, $\Delta\rho$ is the density difference, ρ is the density of the lower layer, g is the acceleration of gravity, ζ is the elevation of the upper surface, and ζ' is the elevation of the pycnocline. In all the above equations and those following, a primed quantity refers to the lower layer and an unprimed quantity refers to the upper layer.

By assuming a time factor of $e^{i\sigma t}$, where σ is the angular frequency, it is possible to eliminate ζ, u, and u' from the resulting equations to obtain the fourth order equation

$$\left[\sigma^4 + \sigma^2\frac{d}{dx}\left(g\frac{d}{dx}\right) + \frac{d}{dx}\left\{g\frac{\Delta\rho}{\rho}h'\frac{d^2}{dx^2}\left(gh\frac{d}{dx}\right)\right\}\right]\zeta' = 0 .$$

The above equation is subject to the boundary conditions that at the coast-line the total transport and the transport in each layer must be zero. Over the coast, the internal waves will be of a progressive nature. At the shelf break, which is essentially a discontinuity in depth, the total transport, the transport in each layer, and the pressure will each be continuous.

1.2.3 Solutions of the Equations

By using the technique of normal mode solutions and by retaining only the lowest power of $\frac{\Delta\rho}{\rho}$, it is possible to separate the fourth order equation into two second order equations that can be solved subject to the boundary conditions listed above.

The solution for the internal wave over a flat shelf is

$$\zeta_i' = \zeta_0 h\left(\frac{1}{d} - \frac{1}{D}\right)\sqrt{\frac{1 + k_2^2 l^2}{\cos^2 k_1 l + \frac{k_2^2}{k_1^2}\sin^2 k_1 l}}\; e^{i(\alpha-\beta)} \cos k_1 x,$$

and the solution for the internal wave seaward of the shelf is

$$\zeta_i' = \zeta_0 h\left(\frac{1}{d} - \frac{1}{D}\right)\frac{k_2 l \cos k_1 l - \frac{k_2}{k_1}\sin k_1 l}{\sqrt{\cos^2 k_1 l + \frac{k_2^2}{k_1^2}\sin^2 k_1 l}}\; e^{i(k_2 l - \beta + \frac{\pi}{2})}\; e^{-ik_2 x}$$

In the above equations ζ_0 = elevation of the free surface due to the surface wave,

$$k_1^2 = \frac{\sigma^2 \frac{d}{h(d-h)}}{g\frac{\Delta\rho}{\rho}},$$

$$k_2^2 = \frac{\sigma^2 \frac{D}{h(D-h)}}{g\frac{\Delta\rho}{\rho}},$$

$$\tan\alpha = k_2 l,$$

$$\tan\beta = \frac{k_2}{k_1}\tan k_1 l.$$

For comparison with the model data, the amplitudes of the expressions for ζ_i' are designated as A over the shelf and B seaward of the shelf. The results of the theory are shown most readily by plotting the ratio of the internal to the surface wave amplitudes against the parameter P, where

$$P = \left(\frac{\sigma^2}{g \frac{\Delta\rho}{\rho} h}\right)^{\frac{1}{2}} l .$$

1.5 Early Model Studies

The idea of using a model to investigate internal waves is old, although not many published results are available. As early as 1912 Zeilon (1912) conducted experiments in a very small model. He was particularly interested in the generation of internal waves over submarine ridges and other bottom features as an explanation for the subsurface wavelike disturbances found in Gullmar fjord. Although his model experiments did not lead to quantitative results, he did demonstrate that low amplitude surface waves passing over a bottom ridge can cause internal waves. Zeilon (1912, 1934) published several photographs of internal waves generated with different conditions of bottom topography.

Early model studies of internal waves were also made by Sandstrom (1908), who investigated the possibility that wind and atmospheric pressure changes generated internal waves. His work contains useful information on the formation of two-layer systems in small model tanks.

In his last paper on internal waves, Zeilon (1934) illustrates qualitatively, by the use of models, that a tide passing over a continental shelf may generate internal waves.

1.3 Purpose of Present Model Studies

Model studies were made because they could provide a large amount of basic information on internal waves in a relatively short time and at a very nominal cost. In a model it is possible to vary the parameters of the problem so that a wide range of conditions may be studied, whereas to observe similar variations in the ocean would probably require many years of expensive field work. Also, the problem can be made much simpler in a model, and the many possible parameters affecting the problem can be handled separately or at least in smaller numbers.

It should be emphasized that there is no general agreement on the most important or even the possible mechanisms of internal wave generation. The demonstration, with a model, that low amplitude surface waves passing over a continental shelf can, in agreement with theory, generate large internal waves strengthens the argument for the importance of this mechanism in the ocean.

CHAPTER 2. EXPERIMENTAL

2.1 Description of the Tank

The experiments were conducted in a wave tank of the Department of Oceanography at the University of Washington. The tank was 3.7 m long, 2.1 m wide, and had a maximum usable depth of 0.4 m. It was desirable to utilize a narrow channel to eliminate cross channel effects and to reduce the subsurface effects of the generator by separating it as far as possible from the area of interest. A channel 0.43 m wide was formed in a U-shape by constructing a wall around the interior of the tank (Figure 1). The wall was lined with plastic sheets, similar to the outside wall, to minimize the effects of lateral friction. Reflectors were placed at the corners of the tank to facilitate the propagation of the surface wave around the bends.

The use of the wide wave tank caused several problems that would not have been present in a long narrow tank. The channel constructed around the edge of the tank was not of a permanent nature, and it was difficult to keep in a uniform width. The necessity of reflecting the surface wave around two 90-degree corners initially caused some problems with cross channel effects. By modifying the size and shape of the reflectors it was possible to eliminate all detectable cross channel effects.

The temporary interior wall caused two additional problems in the conducting of the experiments. The first was that the wall could not be made waterproof because the center basin had to be filled with water. It was relatively easy to open a section of the wall while adding a new upper layer so that the two layer system would be present throughout the tank. There was limited communication between the center and the outside channel; therefore, mixing due to the action of internal waves in the channel caused changes in

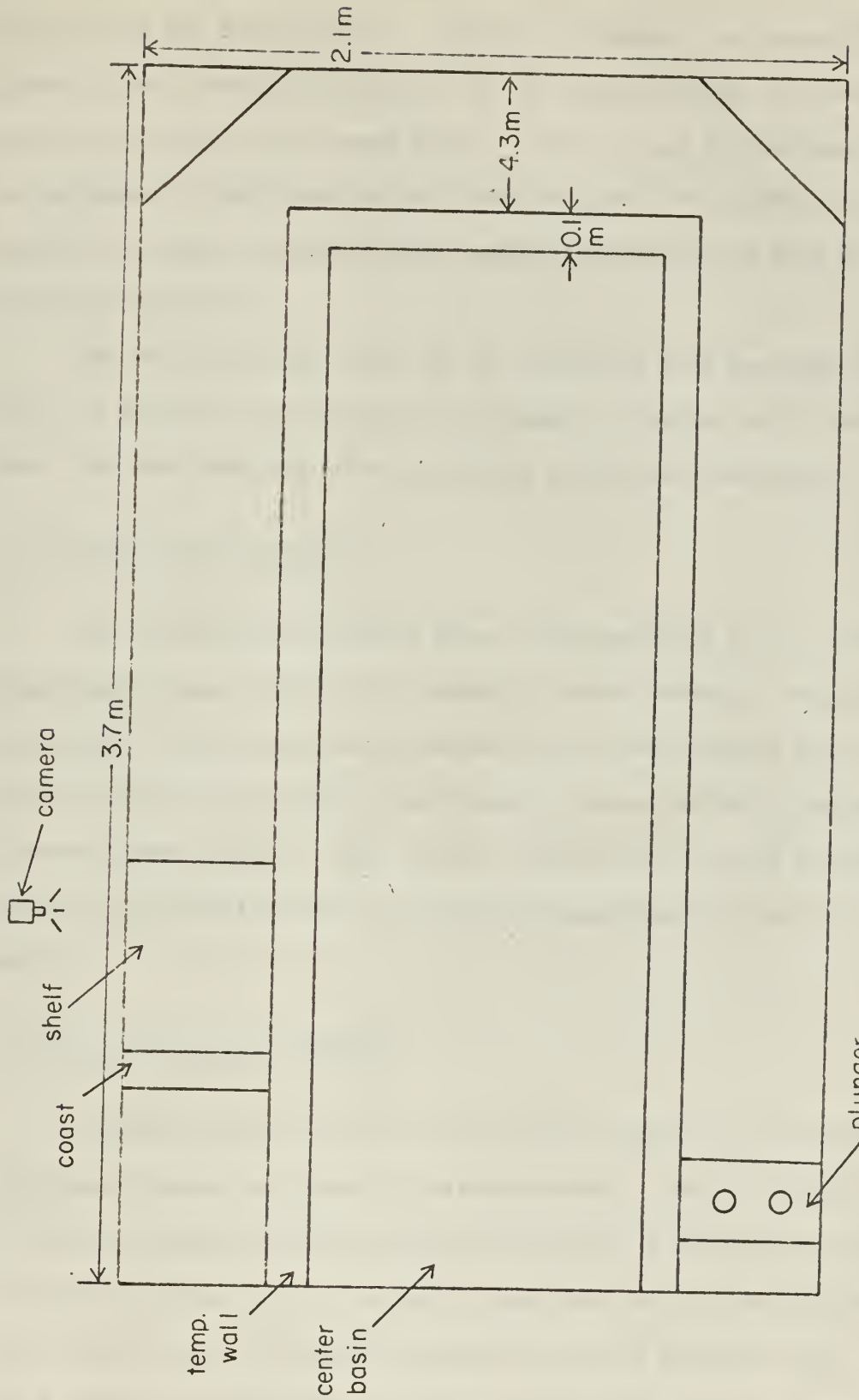

Figure 1. Plan view of model tank.

the water in the center basin. Attempts to sharpen the pycnocline by siphoning were made more difficult by the larger volume of water that was involved because of the center basin. However, all indications were that any influence of the large center basin occurred very slowly and could be neglected for the relatively short period (normally less than an hour) of any series of runs.

The other problem caused by the temporary wall was that the shelf had to be sealed to the sides of the channel. Plastic putty eliminated all leaks, but made changing shelf positions a very slow procedure.

2.2 Surface Wave Generation

The low amplitude surface waves were generated with a vertically oscillating plunger driven by an electric motor through a variable speed transmission, which permitted variation of the wave period over a wide range. The travel and the position of the plunger were adjustable, making it possible to control wave amplitude and, to some extent, the form of the wave. The period of the generated wave was adjusted approximately, and then timed exactly.

2.3 Model Continental Terraces

Although several models of continental terraces were constructed, only simple shapes were used in this experiment. The type used for most of the runs was simply a vertical slope followed by a horizontal shelf (Figure 2). One slight modification of the basic shape was made by rounding off the shelf break in order to minimize mixing caused by turbulent flow. A terrace with a vertical rise and then a sloping shelf (Figure 3) was used, but because friction was apparently important in this situation, and also because the

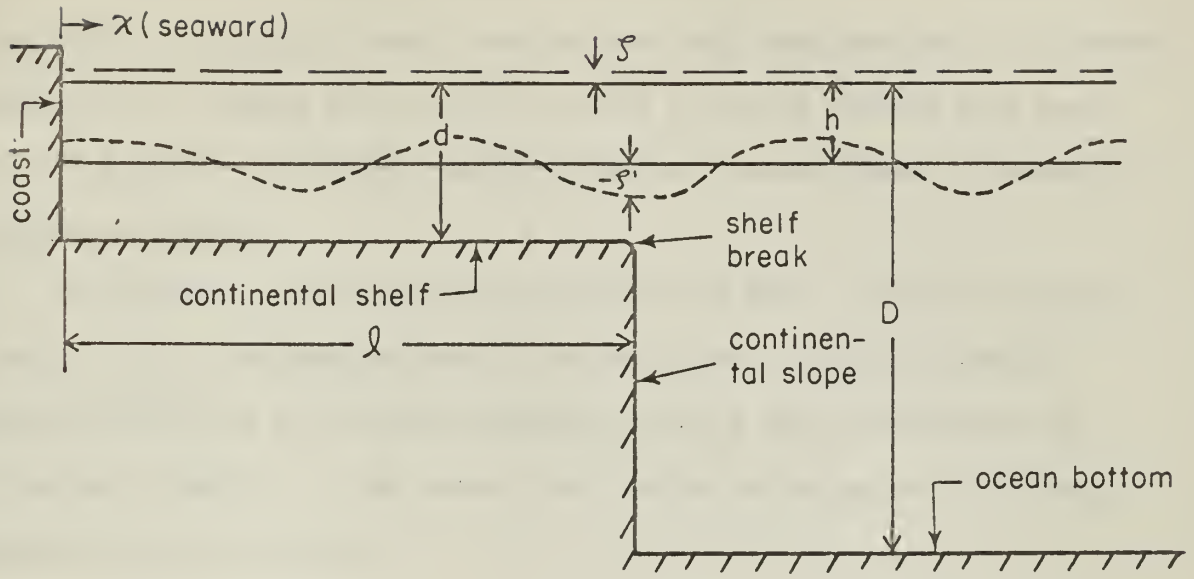

Figure 2. Model continental terrace with flat shelf.

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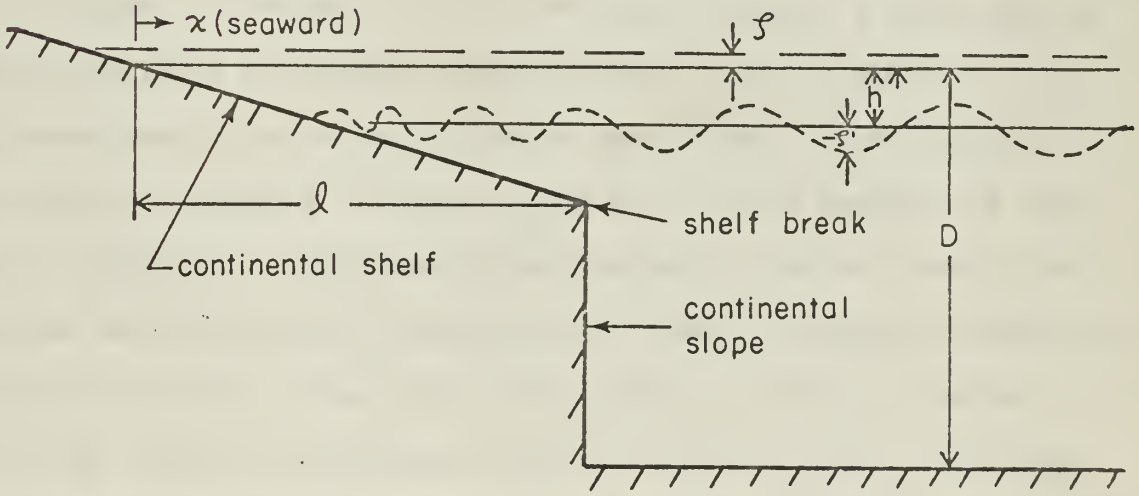


Figure 3. Model continental terrace with sloping shelf.

sloping pycnocline considered in Rattray's theory could not be maintained in the model, this type of shelf was examined only qualitatively. A terrace consisting of a sloping plane with the plane at angles ranging from about 5° to 17° from the horizontal was tried, but no internal wave of measurable amplitude was formed.

The vertical continental slope used in the model does not correspond to nature, but is an idealization of the condition in which the depth changes rapidly over a horizontal distance that is small compared to an internal wave length. In the ocean a continental slope of about 30° would approximate this condition.

2.4 Formation of a Two-Layer System

A layer of fresh water above a layer of salt water was used for the two-layer system. Formation of such a two-layer system in a large tank is difficult, but after preliminary experimentation it was possible to form sharp pycnoclines and to maintain them for several days. The principal consideration in forming a two-layer system with a sharp boundary is that the upper layer must be introduced without mixing it with the lower layer. To minimize the mixing it is necessary not to create horizontal currents and to introduce the water at the level of the surface, so that no vertical motion of the introduced fresh water occurs. Neither of these conditions can be met, but by adding the fresh water very slowly a minimum of turbulent mixing takes place. Five U-shaped manifolds were constructed of metal tubing, and many small holes were drilled along the flat bottom of the U. The line of holes was placed at the surface of the lower layer and water was allowed to flow slowly through the manifolds. Two typical density profiles are shown in Figure 4 where curve A represents the profile soon after the adding

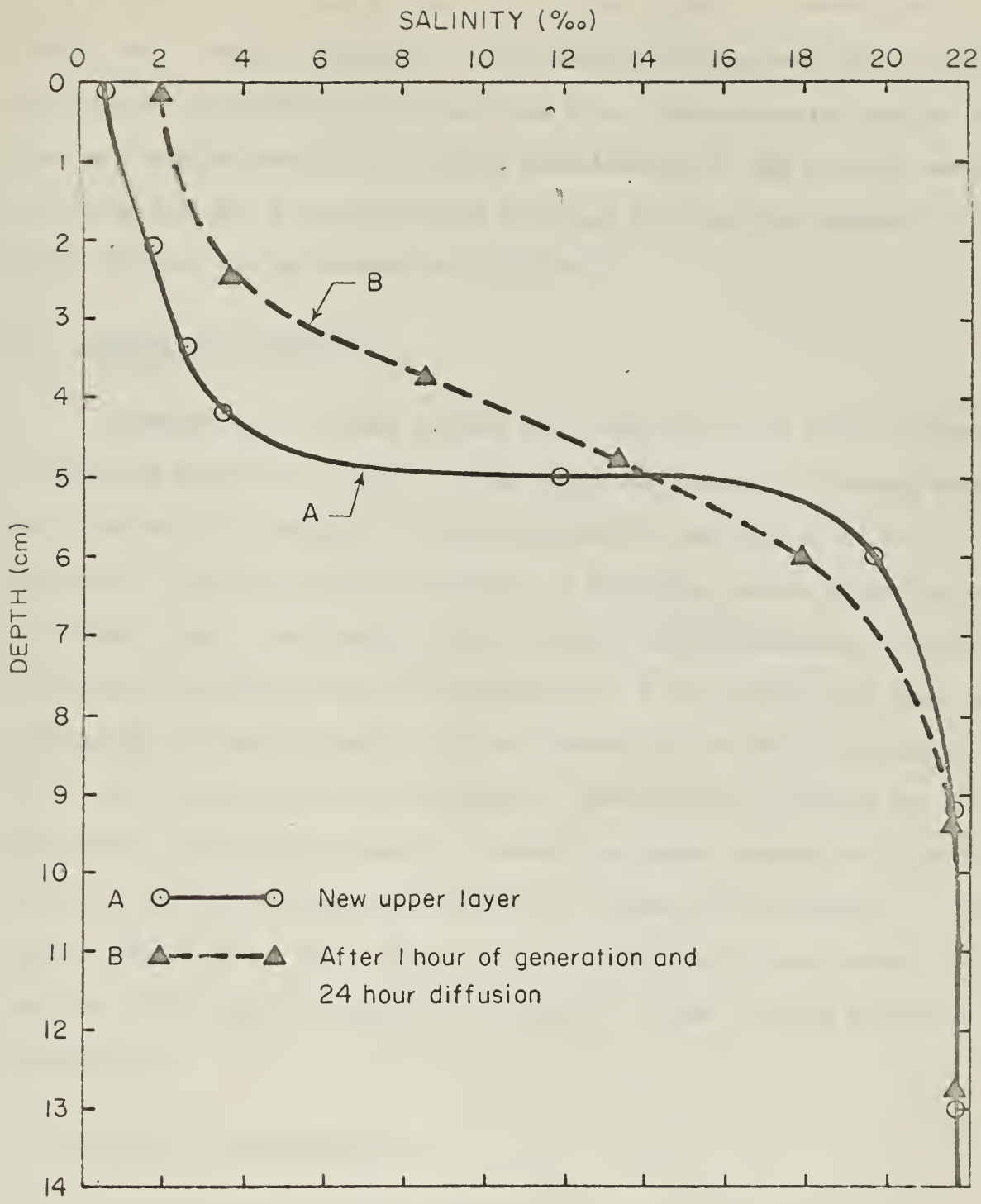


Figure 4. **Salinity profiles for new and old upper layers.**

of a new upper layer, and B, the profile after 1 hour of internal wave action and 24 hours of diffusion. The density profiles were checked before each run and immediately after most runs with a micro-salinity sampler that drew six samples from selected depths simultaneously. The salinity samples were titrated with a microtitration technique that had been tested in other model work and had an accuracy of 0.05 °/oo S.

2.5 Marking the Pycnocline

Photographic recording methods were used; therefore it was necessary to mark the pycnocline clearly. A dye strip was formed by injecting methyl red solution, intermediate in density between the two layers, at the pycnocline. The injection was made with a hypodermic needle guided by an adjustable track. The single narrow dye strip had the advantage of being removable by siphoning with the manifolds or, if given sufficient time, of dissipating through diffusion. Another possibility was to completely dye one of the layers and use it as a marker. When one of the layers was dyed completely, the interface tended to become indistinct because of diffusion or mixing, and it was much more difficult to clear it by siphoning. It was also necessary with a fully dyed layer to observe the internal waves from a position exactly perpendicular to their path in order to avoid distortion by parallax.

2.6 Methods of Collecting Data

2.6.1 Photographic Measurement Method

A photographic method of data recording was utilized because it satisfied the requirements of low cost, accuracy, and availability. By

operating a 16 mm motion picture camera at low speed (8 frames per second) it was possible to get sufficient accuracy at a very low cost. Good results were also obtained with a 35 mm rapid sequence camera, but it had no real advantage over the movie camera. A grid of length scales was marked on the plastic side of the tank, and a reference clock was mounted near the area of interest. Some difficulty was caused by the short distance between the tank and a wall of the room, but by using a short focal length lens good results were obtained. A small amount of distortion did occur but could be corrected by comparison with the similarly distorted reference lines.

The main disadvantage of the photographic data recording system was the time lag between taking the pictures and obtaining the data from them. Because it was difficult to determine the properties of the internal wave by observing them in the model, it was necessary to make many more runs than would have been necessary if the results could have been viewed immediately after each run. Moreover, many runs that appeared successful did not turn out well, and there was no opportunity to repeat all of the unsuccessful runs.

2.6.2 Surface Wave Measurement

The determination of surface wave height was the most difficult measurement. The first method attempted was to utilize a device developed to measure tidal heights in a model. It consisted of a float attached to a balance with a visually read scale. Several difficulties were encountered including slow response time, a tendency for the needle to oscillate considerably after the wave had passed, and difficulty in reading the dial because the surface wave passed very quickly. No modifications were found that solved all of these difficulties without introducing others.

Because the principal difficulties were due to the low amplitude and the rapid transit of the surface waves it was decided that photographic measurements would be the most suitable. A meter stick placed in a vertical position above the water just inside the plastic side of the tank served as a reference to determine the height of the surface waves. A small strip of dye, less dense than the tank water, was placed about 1 cm from the edge of the tank to eliminate boundary effects. The 16 mm camera, placed approximately 25 cm from the side of the tank, was then run at a high speed (64 fps) while several surface waves passed the field of view. Later the pictures were projected on a screen and the surface wave amplitudes were measured.

The surface wave measurements are the least accurate data from the experiment; some of the sources of error are refraction from the water adhering to the side of the tank, difficulty in measuring the small waves even when projected on a screen, and error in interpolation because all of the surface waves were not measured for each run. Most of these errors were small, and by testing interpolated values against measured values, it was found that all errors were less than 15 percent.

2.6.3 Internal Wave Measurement

Measurement of internal wave characteristics was much easier than that of the surface waves, because the internal waves were generally much larger and slower moving. By photographing each run and later projecting the movie or film strip on a screen, it was possible to get good measurements of amplitude, wave length, and any other visible characteristic. Considerable care was exercised to project the image squarely onto the screen so that no distortion was introduced. Also, tracings were made by attaching large

sheets of paper to a wall and tracing the projected wave image and reference marks. Direct measurement of the internal waves was attempted in the early experiments, but it was found that the results were highly variable because the internal waves were small and in motion.

2.6.4 Pycnocline Measurements

The exact thickness of the upper layer and the sharpness of the pycnocline were determined by plotting the salinity values versus the depth, and then connecting the points with a smooth curve (Figure 4). In most cases the thickness of the upper layer was obvious because the salinity profile was nearly discontinuous, but in cases where there was any doubt, the depth where the salinity was intermediate between that of the upper and lower layer was used.

2.7 Direct Measurements from the Model

Several parameters were measured directly from the model. The depth of the water over the shelf, the depth seaward of the shelf, the shelf width, and the slope of the shelf were measured with calipers and a meter stick. The period of the surface wave was measured exactly by using a stop watch to time 5 or 10 revolutions of the generator drive wheel.

CHAPTER 3. OBSERVATIONS AND RESULTS

3.1 General Observations

It was observed early in the experiment that internal waves were readily formed over a wide range of conditions by the low amplitude surface wave passing over the shelf break. Since the values of l, h, D, σ, and $\Delta\rho$ were not critical, it was apparent that this mechanism of internal wave generation was very effective.

Another early observation was that the internal waves traveled seaward from the shelf break for long distances with only slight dissipation. There appeared to be more dissipation as the pycnocline weakened, but rates of decay for different density profiles were not determined.

The general nature of the internal wave formation is shown quite clearly in the still photographs of Plate I. The photographs in Plate I illustrate the relatively large amplitude of the internal waves, the smaller internal waves normally found over the shelf, and the breaking of the internal waves that sometimes occurred near the shelf break. The negative prints show standing waves which sometimes formed over the shelf, and internal waves when the pycnocline is below the shelf. The negative prints were made from the movie films and are not as clear as the positive prints from the 35 mm films, but they are included to show features that could not otherwise be shown.

The model studies did not permit accurate determination of phase relations between surface and internal waves. The photographic method would seem to be well suited to phase determination, but the surface waves were of such low amplitude that it was necessary to move the camera in very close to measure them, and with the camera in close, the internal waves were not visible. A short focal length lens would simplify the phase determination.

3.2 Internal Waves Over the Flat Shelf

Three different internal wave conditions were observed over the flat shelf. Standing waves over the shelf, as predicted by Rattray's theory, were observed when friction was not important. The second condition, which was more common in the model was to have progressive internal waves over the shelf.

The third condition, in which there were breaking internal waves appeared to occur when a limiting amplitude was reached. The breaking was observed as a pinching-off of the first crest or trough of the wave as it formed, and took place when the value of σ was large and P was greater than 9. The wave proceeding up the shelf was the same shape as the non-breaking wave but had a somewhat smaller amplitude (Plate I, photograph 9).

3.3 Internal Waves Over a Sloping Shelf

Internal waves generated over a sloping shelf differed from those generated over a flat shelf because friction and breaking were always important. As the internal wave traveled shoreward, the effects of the bottom increased. The internal waves decreased in amplitude and in wave length as they moved up the shelf. In those cases where the pycnocline intersected the sloping shelf, the internal waves steepened as they moved up the shelf until they appeared to reach an unstable configuration and break. However, observation was difficult because the horizontal oscillation of the whole water mass caused by the surface waves partly obscured the breaking. There were no standing waves on the continental shelf because there was little or no reflection from the coastline. The sloping shelf is considered in Rattray's (1960) theory, but he assumes that the pycnocline

also slopes over the shelf. Because it was impossible to maintain a sloping pycnocline in the model and because friction and breaking were important, quantitative measurements were concentrated on the flat shelf model. The sloping shelf is an important condition, and it would be useful to conduct further experiments to investigate this case more fully.

3.4 Internal Waves Seaward of the Shelf

The internal waves traveling seaward from the shelf break were progressive for both the flat and sloping shelf conditions. The characteristics of the internal waves were in general accord with theory with only minor variations that will be discussed quantitatively in the next chapter and qualitatively in the succeeding paragraph.

The form of the internal waves over the continental shelf will influence the internal waves traveling seaward from the shelf break. When there is a standing wave over the shelf, it is formed by a progressive wave traveling from the shelf break to the coast, reflecting from the shore, and traveling back to the shelf break. If the wave continues to travel seaward, it will interfere with the internal wave being formed at the shelf break and also traveling seaward. The effect of the interference will then be determined by the phase relationship which, in turn, will be determined by the shelf width. When the interference is completely destructive no internal waves will be detectable seaward of the shelf, which is the case corresponding to the points where the theoretical curve intersects the P axis in Figure 5. This completely destructive interference was never observed in the model, perhaps because an insufficient number of standing waves were observed.

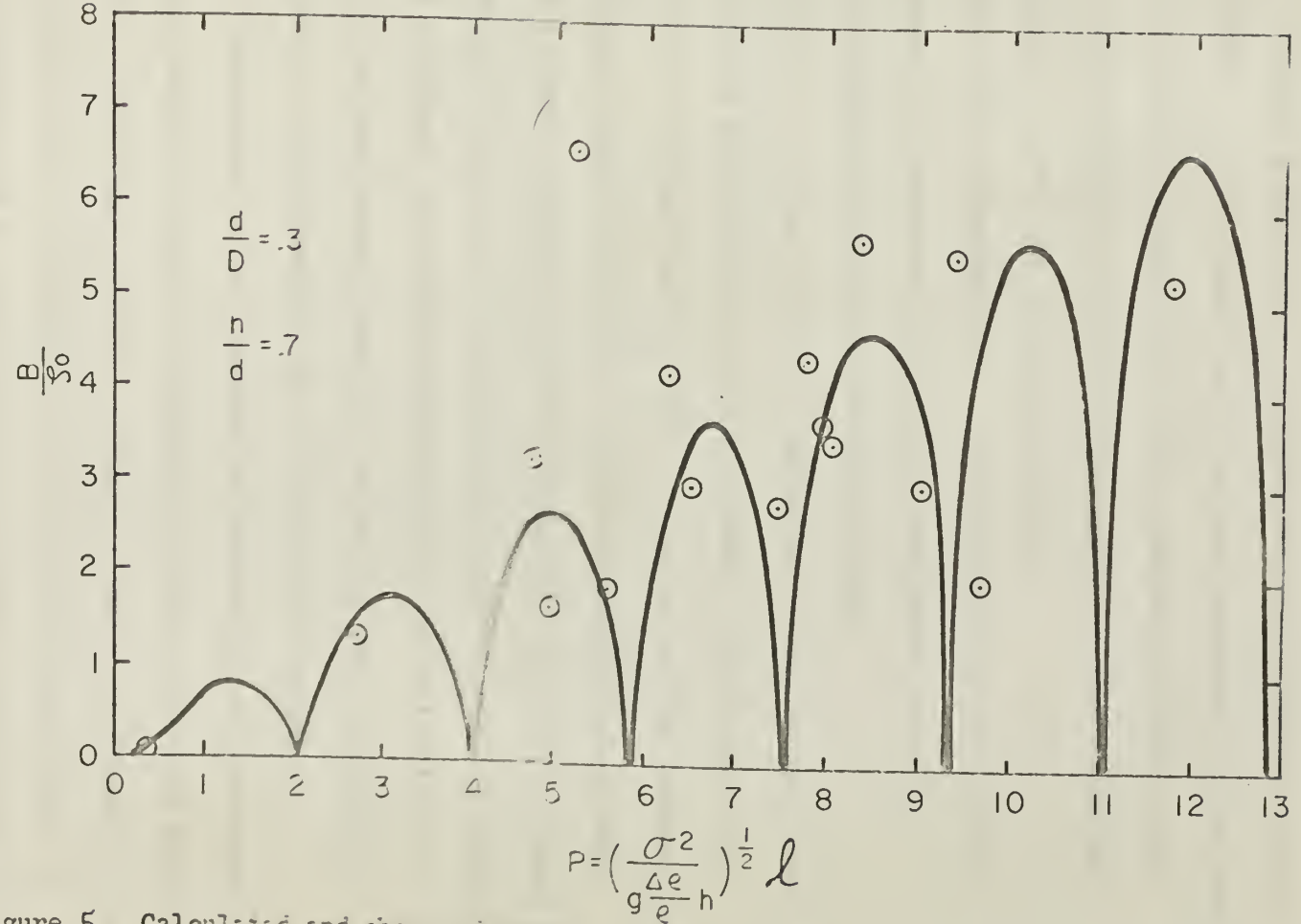


Figure 5. Calculated and observed internal to surface wave ratio seaward of the shelf for $h \gtrless d$.

3.5 Mixing Caused by Internal Waves

Visual observations of the disappearance of the dye line in the vicinity of internal wave generation showed that rather violent mixing was taking place. In Plate I this mixing is indicated by the fainter dye lin near the shelf break. In all cases some mixing was detectable by the fading of the dye line along the entire length of the internal wave train, but particularly strong mixing was observed in the region of wave formation when the pycnocline was near the depth of the shelf.

The mixing that is possible under some circumstances is shown in Plate I, photograph 12, where two lines of dye were inserted at the pycnocline and the photograph taken a few minutes later.

In addition to the slow effects of diffusion, mixing brought about by the internal waves caused the pycnocline to weaken. The pycnocline became spread over a vertical distance of 2 or 3 cm rather than remaining an almost discontinuous change in density. It was recognized that it would not be possible to handle quantitatively the generation of internal waves in this case, although Fjeldstad (1935) has developed a theoretical treatment of such density profiles. An experimental difficulty is an inability to reproduce exactly the same density profile, but in any case Fjeldstad's theory is difficult to apply usefully to the conditions studied.

General observations in the presence of weakened pycnoclines showed that the internal waves dissipated more rapidly and were of smaller amplitude. This effect was not extremely important, although it made conditions for the formation of internal waves more critical. It was also observed that the internal waves traveled with a lower velocity, in agreement with Zeilon's (1913) paper, although the decrease in velocity was small.

3.6 Currents Induced by Mixing

Steady currents produced by mixing due to internal waves were unexpected results of the experiment. In some of the runs considerable mixing took place near the shelf break, and subsequently the line of dye would be carried out away from the shelf. By using a small piece of cork as a float, an appreciable surface current, flowing in towards the shelf was detected. Evidently, the mixing taking place in the vicinity of the shelf was adequate to form a layer of water intermediate in density between the top and bottom layers, which then ran out along the pycnocline. The surface current was evidently a compensating current, and it would be expected that there would be a corresponding current in the bottom layer.

3.7 Supplementary Motion Picture

As a supplementary part of the model studies a 14-minute motion picture was produced to demonstrate the formation of internal waves by low amplitude surface waves passing over a continental shelf. The motion picture was made with the same experimental arrangement discussed in Chapter 2, except that the movie camera was run at regular speed.

Several effects were illustrated in the motion picture by varying individual parameters in large increments. The effect of increasing the surface wave amplitude was shown to be a proportional increase in internal wave amplitude. A severely shortened shelf permitted the formation of only small amplitude internal waves. It was demonstrated that an upper layer thicker than the depth of water over the shelf still permitted large internal waves; in this case there was then no interference from a reflected internal wave from over the shelf. It was shown that if the lower layer was

still fairly thick over the shelf, friction was not important and a standing wave was formed over the shelf. Also, when the pycnocline was nearer the depth of the shelf, the internal wave over the shelf was dissipated by friction and would not form a standing wave.

Several variations of shelf slopes demonstrated the increasing steepness of the internal wave and the decrease in wave length. The effect of friction as the wave moved up the shelf could be detected, because the internal wave did not increase in amplitude as it approached the intersection of the shelf and the pycnocline. In some instances it was possible to see the internal wave break as it moved up the shelf.

The motion picture also illustrates that in estuaries and polar regions a current system might be established by the mixing at the shelf region. As was discussed in the previous section, a surface current flowing in toward the shelf and a seaward current at the pycnocline could be detected.

Mixing in the area of internal wave generation could be observed in the movie because of the difference in the refractive index of the two layers and the diffusion of the dye lines.

CHAPTER 4. DISCUSSION

4.1 Modal Conditions Compared to Oceanic Conditions

There are several factors of internal wave formation that should be considered in the ocean that were not important in the model. The pycnocline in the ocean would not normally be sharp, and its form might vary with the horizontal coordinates. The shelf and coast would not be of the idealized form used in the model and many different effects would be possible. Breaking of internal waves may be more common in the ocean than in the model, because the density differences between layers are generally much smaller than in the model. Observations of breaking internal waves have been made in the ocean, particularly in the Straits of Messina and in the "internal surf" found off the west coast of Africa (Defant, 1950).

An interesting case is that in which friction is of moderate importance but where no breaking occurs, so that the character of the internal wave should change over the shelf. The internal wave should be a standing wave near the shoreline, where the effects of friction are felt only over a short distance, but farther out on the shelf the internal wave would be a progressive wave, because friction would act over a longer distance.

4.2 Comparison of Model Results with Theory

4.2.1 General Comparisons

The results of the experiment were compared with the theory of Rattray (1960) in the case where model conditions were most similar to those assumed in the theory, although the conditions could not exactly duplicate those assumed in the theory.

It was necessary to use rather large density differences in the model, as explained previously. The theory neglects higher powers of $\frac{\Delta\rho}{\rho}$, and hence a slight deviation from theory may be expected, although $\frac{\Delta\rho}{\rho}$ was at most 0.03.

The theory assumes a two-layer system, but the model always had some finite thickness to the pycnocline. Two typical density profiles (Figure 4) have been discussed in previous chapters. There should not be much difference between the sharp pycnocline and a two-layer system.

Friction is not included in the theory, whereas both bottom friction and internal friction were present in the model. Internal friction did not have a detectable effect on the model experiments, but the effect of bottom friction was important under certain conditions. It was found that the effects of friction could be minimized by making the ratio $\frac{h}{d}$ as small as practical. Utilizing small values of $\frac{h}{d}$ was not always possible in the model because the depth of the tank was only 40 cm, and it was necessary to make $\frac{d}{D}$ and h large enough to get internal waves of sufficient amplitude for accurate measurement. Most of the experiment was carried out with $\frac{h}{d} = 0.7$, a value large enough that friction became important over the upper half of the range of P studied.

The theory lead to such complicated curves, in most cases, that comparison with measured values is difficult. A small variation in the value of P corresponds to a wide range of $\left|\frac{A}{J_0}\right|$ or $\left|\frac{B}{J_0}\right|$. It is evident, particularly in Figure 5, that a small variation in P near a zero of the curve can give a large difference in the value of $\left|\frac{B}{J_0}\right|$. In this experiment, many measurements are represented in each point obtained from the model. (See Appendix III which contains the most reliable experimental data.) To obtain the value of

P, measurements of σ, P, h, and l are involved, and to get $\left|\frac{A}{\zeta_0}\right|$ or $\left|\frac{B}{\zeta_0}\right|$, measurement of A or B and the difficult measurement of ζ_0 are necessary.

4.2.2 Variation of Internal Wave Amplitude with Surface Wave Amplitude

The expression for the internal wave amplitude, B, given in Section 1.4.3 states that the internal wave amplitude should be directly proportional to the surface wave amplitude if the other parameters are held constant. The agreement between the theoretical relationship (Figure 6, solid line) and the measured values from the model (Figure 6, circled points) is quite good, although only a few measurements could be obtained with the other parameters held constant. The tendency of the measured values to fall below the theoretical curve is not definitely significant, but might be the effect of friction.

4.2.3 Comparison for $h > d$

When the thickness of the upper layer is greater than the depth of water over the shelf, the theory predicts a simple relationship of the ratio of the amplitudes of the internal and surface wave, $\left|\frac{B}{\zeta_0}\right|$, to the parameter P. This condition eliminates the internal wave over the shelf and the interference of the internal wave over the shelf with the wave traveling seaward. Figure 7 is a plot of $\left|\frac{B}{\zeta_0}\right|$ against P which shows the theoretical curve (a solid line) and the measured values. The measured values do not coincide with the theoretical curve as closely as would be expected for this relatively simple case, but considering the range of possible errors in measurement, the agreement is reasonably good. It would be desirable to obtain more measured values for these particular values of $\frac{h}{d}$ and $\frac{d}{D}$ over a wide range of P.

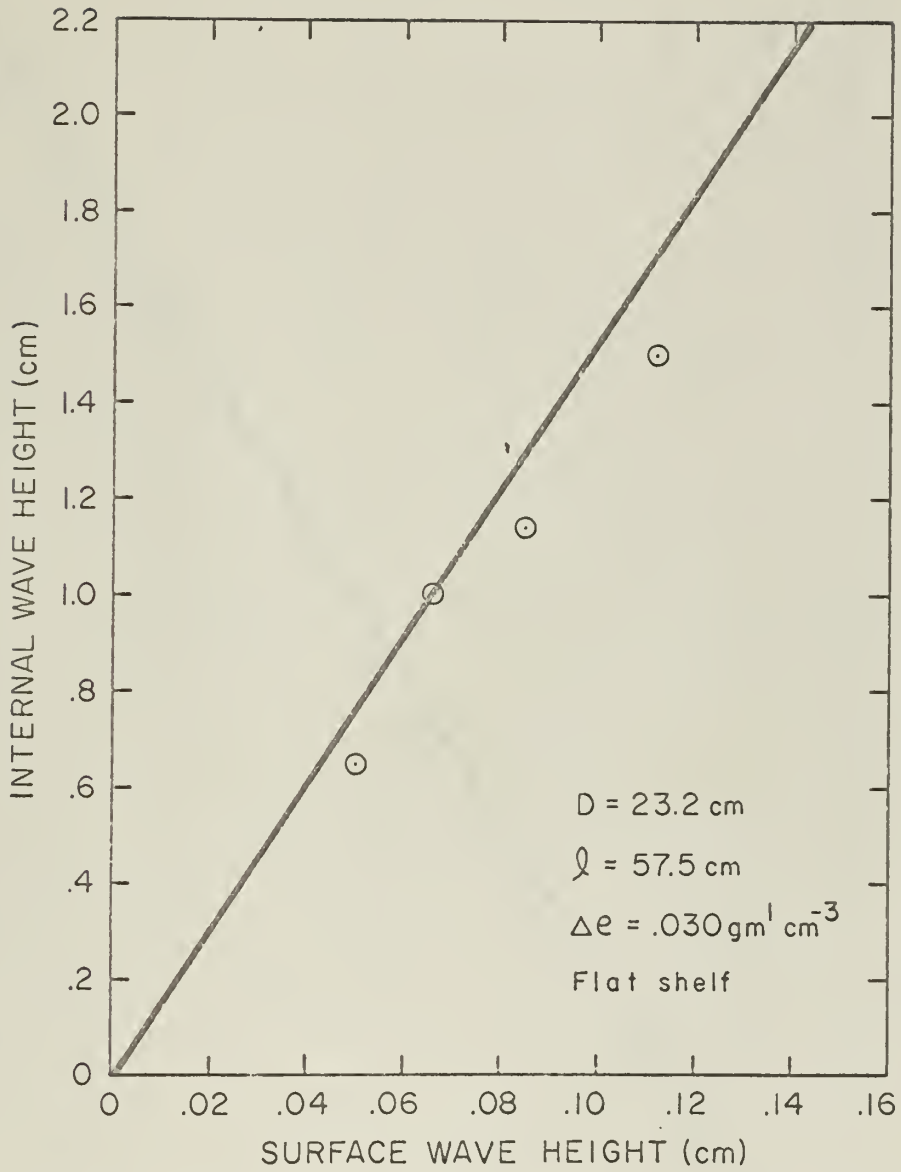


Figure 6. Internal wave height plotted against surface wave height.

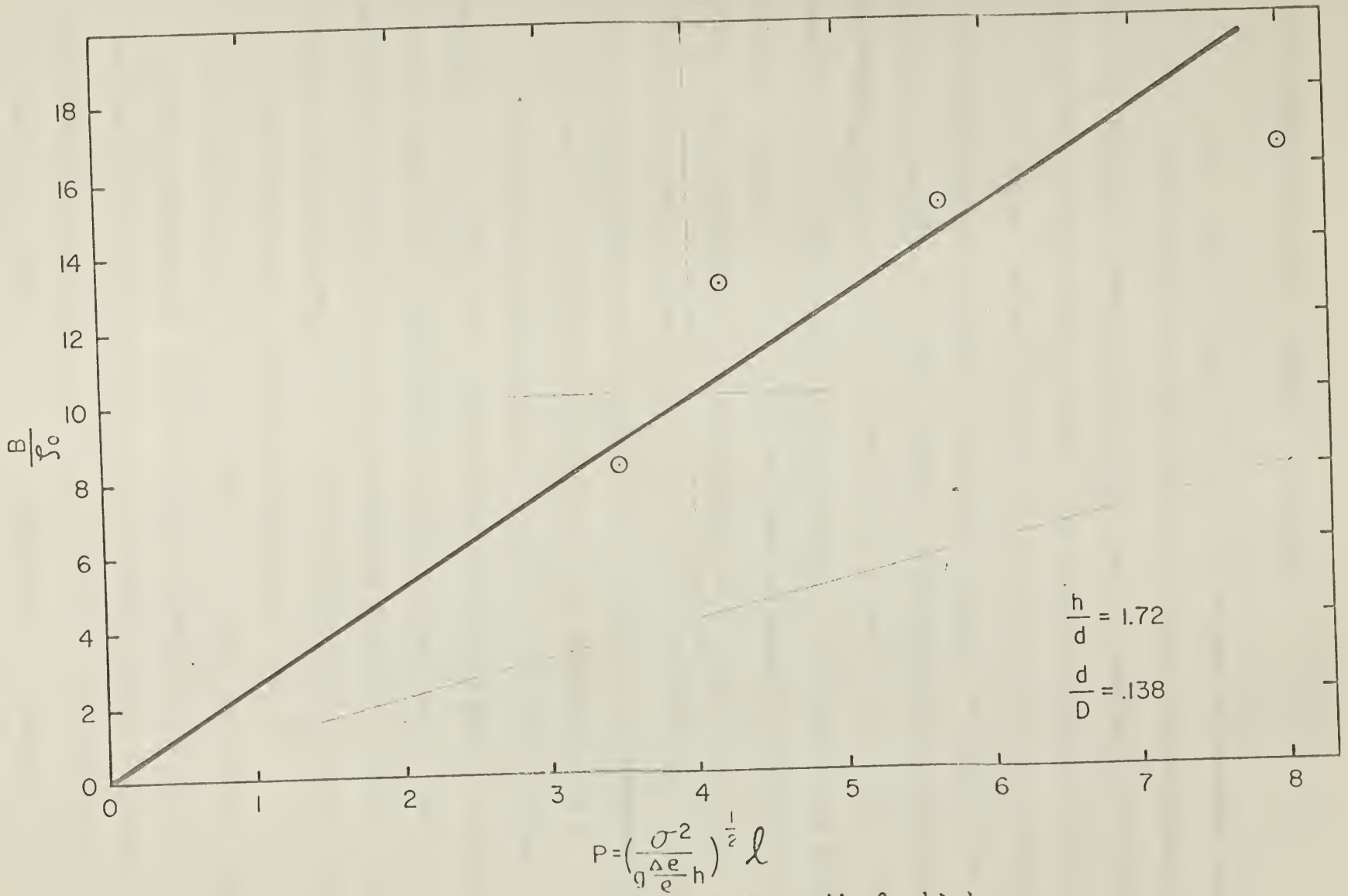


Figure 7. Calculated and observed internal to surface wave ratio for h> d.

4.2.4 Comparison for $h < d$, Seaward of the Shelf

The solid line in Figure 5 shows the theoretical relationship between the ratio of the internal wave amplitude to the surface wave amplitude $\left|\frac{B}{\zeta_0}\right|$ and the parameter P, seaward of the shelf, when the upper layer is thinner than the depth of water over the shelf. The measured values are shown by the circled points. The points lying above the curve should be explained because the influence of friction would not be expected to increase the internal wave amplitude. Careful checks of the recorded data do not show any obvious errors, but for the point lying farthest from the curve it is possible that the shelf width was recorded wrong. At the time this measurement was being made the shelf width was being increased or decreased by a factor of 2. If the value of P were doubled, as it would be if l were doubled, then the point would fall on the curve. The variation of the other points is not so great and is probably due to errors in the measurement of ζ_0 which was very difficult and some error in the measurement of B.

In general the measured values on Figure 6 fall within the range predicted by the theoretical curve and follow the small scale features of the curve quite well, considering that errors in measurements cause an uncertainty of about 15 percent in the ordinate and about 5 percent in the abscissa. The points occurring at values of P where the theoretical curve is at or near zero do not coincide with the curve. This deviation is probably due to friction being important in the model, and the zero points of the curve correspond to the situation where there is destructive interference between the progressive wave traveling away from the shelf and the reflected wave returning seaward over the shelf. If friction were considered in the theory, it would be expected that the maximum values would

be somewhat lower, and that the curve would not go to zero except at the origin.

4.2.5 Comparison for $h < d$, Over the Shelf

The theoretical relationship between the ratio of the internal wave amplitude to the surface wave amplitude $\left|\frac{A}{\zeta_0}\right|$ and the parameter P, over the shelf, is shown as a solid line in Figure 8. In the previous chapter the effect of friction on the internal waves for larger values of P was discussed. The measured values from the model are shown as points, and the agreement with theory is poor.

The more common progressive forms of internal waves over the shelf in the model result mainly from the effects of friction. A standing wave is formed by the interference of two progressive waves of equal amplitude and period traveling in opposite directions. The phase of the two progressive waves is not important, so a standing wave will be formed if the progressive wave formed at the shelf break, travels the length of the shelf and is reflected, without dissipation. The factors tending to cause rapid internal wave dissipation prevent the formation of standing internal waves. These factors include a wide continental shelf, a shallow bottom layer over the shelf, and internal waves of short wave length. It is also important for standing wave formation that the reflection be without dissipation, which was the case when the coast was a vertical wall.

The average measured values of $\left|\frac{A}{\zeta_0}\right|$ decrease slightly with increasing P while the predicted values increase rapidly. Qualitatively, the difference can be explained by the effects of friction. Upon examining the effects of parameter $P = \left(\frac{\sigma^2}{g^{\frac{1}{2}} h}\right)^{\frac{1}{2}} l$ it is apparent that σ and l are the parameters

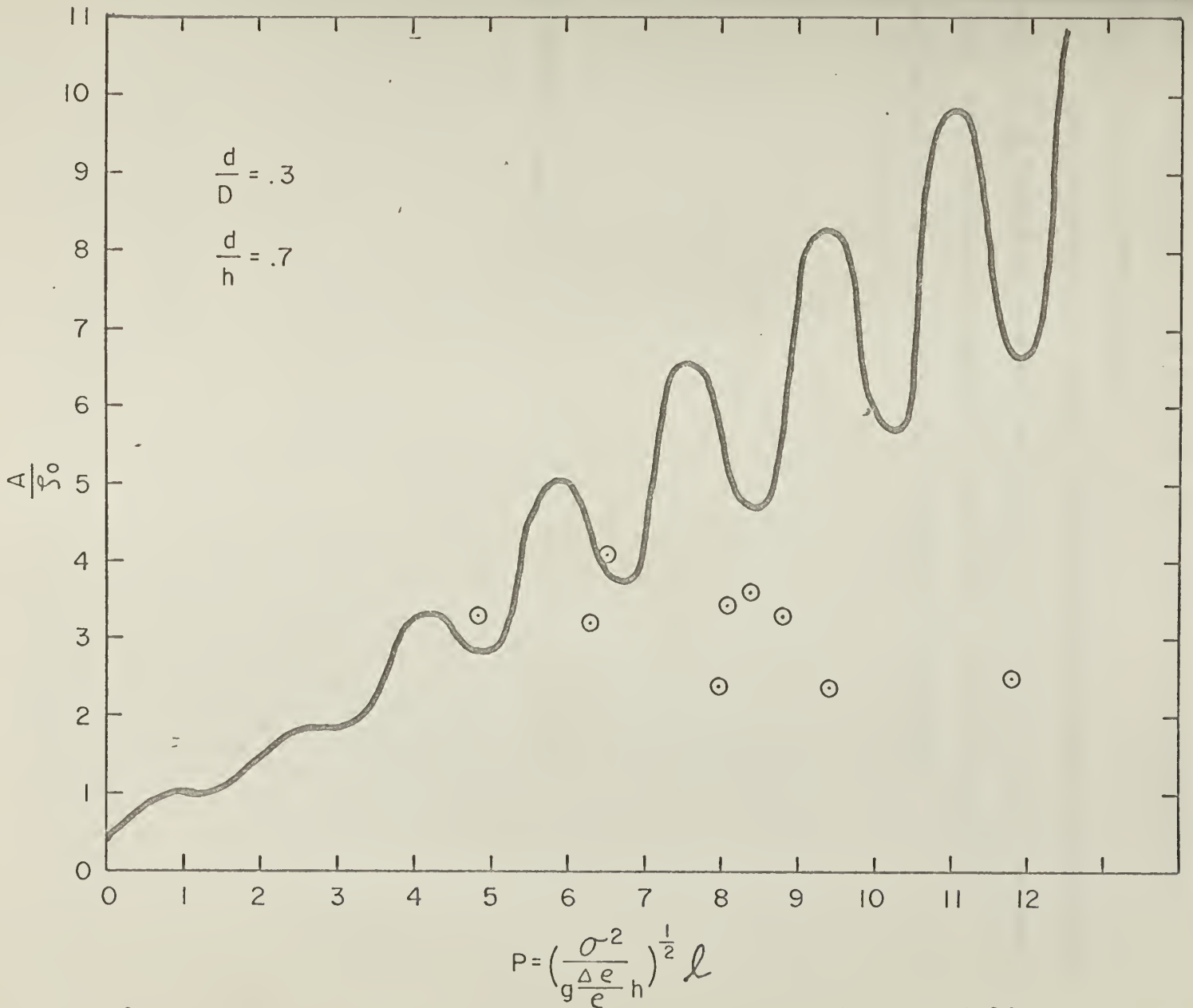


Figure 8. Calculated and observed internal to surface wave ratio over shelf.

most widely varied in order to obtain different values of P in the model. Increasing the value of l gives a longer distance over which frictional forces act and decreases the internal wave amplitude. Increasing σ will tend to increase velocities near the shelf, and will cause friction to become relatively more important.

When friction is important, it would be expected that the theoretical value of $\left|\frac{A}{S_0}\right|$ would not increase as rapidly with P, as in the frictionless case. When friction is important it would lead to more complicated expressions for the x-dependence of internal wave amplitude.

CHAPTER 5. CONCLUSIONS

5.1 Utility of Internal Wave Model Studies

A low amplitude surface wave traveling over a continental shelf break has been demonstrated to be an effective mechanism for forming large amplitude internal waves in a model. It has been possible to obtain quantitative data from these studies, indicating that the model is a useful tool in the study of internal waves.

5.2 Agreement with Theory

There was generally good agreement between the model experiments and Rattray's theory, although there were departures in detail. The amplitudes of the internal waves formed when friction was not important are in general agreement with the theoretical values, and the variation of the measured values of $\left|\frac{A}{S_0}\right|$ and $\left|\frac{B}{S_0}\right|$ with P is similar to that predicted by theory. The restricted depth of the tank permitted only a limited number of measurements in which the lower layer was thick enough to permit the formation of standing waves over the continental shelf, but for smaller values of P the measurements agreed quite well with the theory. It was found that friction was often important in the model, and in certain cases where the pycnocline is near the bottom of a wide shelf, friction would also be important in the ocean.

5.3 Mixing and Currents

Intense mixing was found in several cases and was effective in causing steady currents. It is possible that similar processes could be important in the ocean.

5.4 Desirable Equipment Improvements

Certain modifications of the equipment are desirable. A long straight tank of greater depth would eliminate the problems with the center basin and with the limited depth that were discussed above. If both sides of the channel were permanent, it would be possible to construct shelves that could be adjusted remotely by cranks without disturbing the two-layer system.

The plunging type generator was generally satisfactory, although it would have been better had the generator been readily adjustable to the exact period desired.

The greatest improvement needed in the experimental equipment would be the development of a data gathering system which would avoid the long delay inherent in the photographic method and in the titration procedure. It might be possible to adapt electronic devices that could be read directly from a recorder.

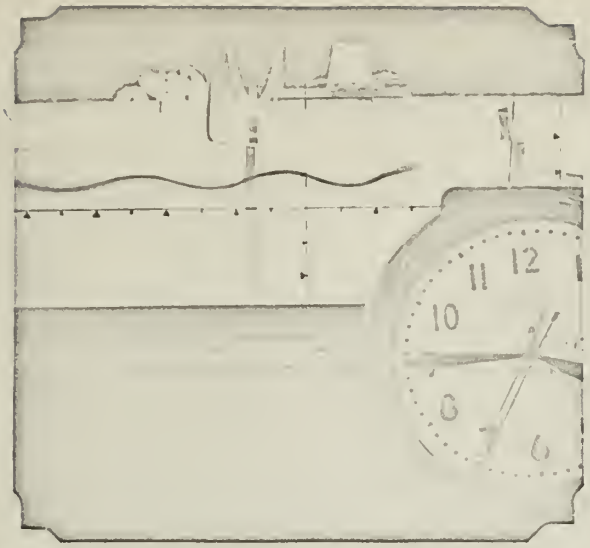
5.5 Possible Extensions of the Experiment

There are several possible extensions of model experiments of this type. Techniques have been developed for obtaining almost any type of stable density profile, given sufficient time. It would be useful to study three-layered systems and density profiles that varied continuously with depth. Continental terraces modeled on actual bathymetric profiles of coastal areas of interest could be used, and it should be possible to predict locations of oceanic internal waves.

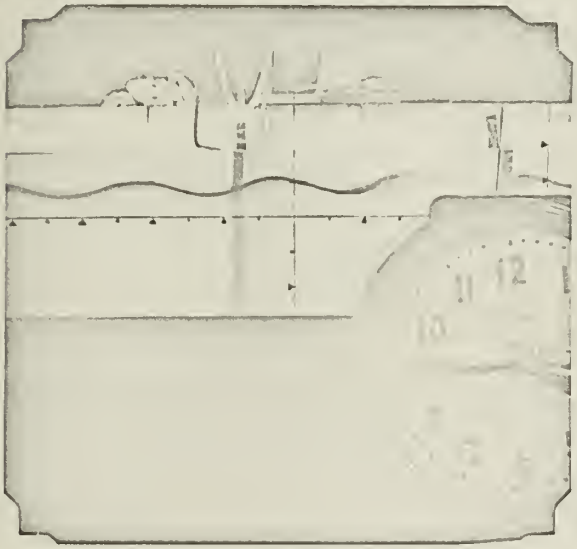
The rate of mixing caused by internal waves in their normal non-breaking form could be investigated in the model. It would be difficult to get similarity, because the lengths and velocities are so much different in

the model, but it should be possible to obtain information that would be a definite advance over the present state of knowledge. With the same limitations, it should also be possible to study the rate of decay of internal waves.

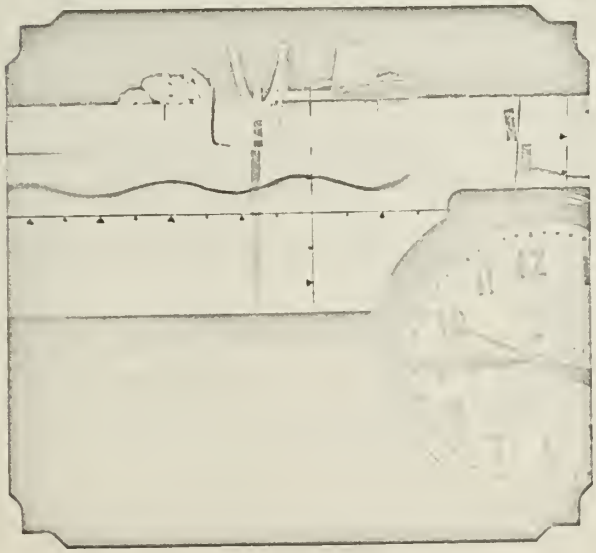
PLATE I



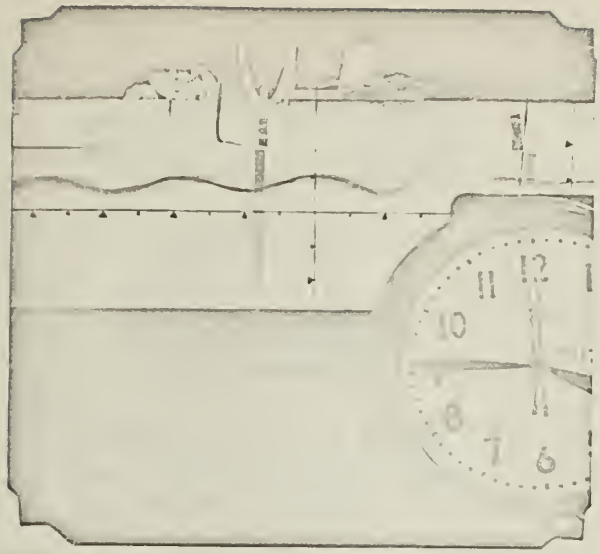
1. A typical internal wave of large amplitude (The shelf break is over the 11 on the clock.)



2. Mixing over the shelf has made the dye line less distinct.

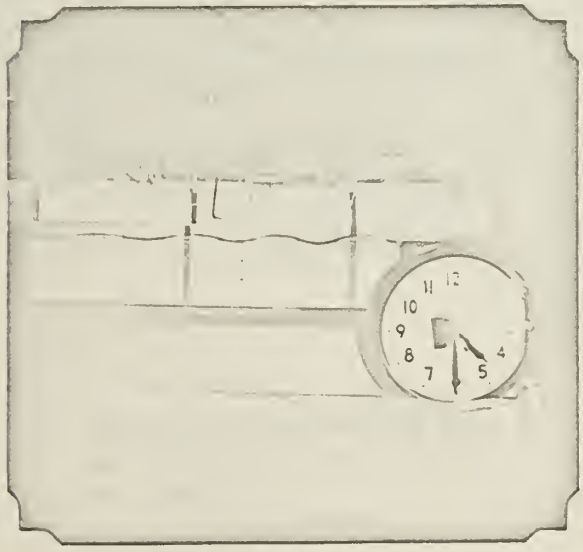


3. More mixing has occurred. (Note the small internal wave amplitude over the shelf.)

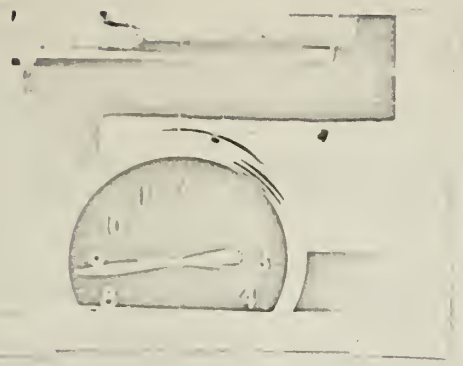


4. A photograph from the same series as 3, showing the smaller amplitude internal waves over the shelf.

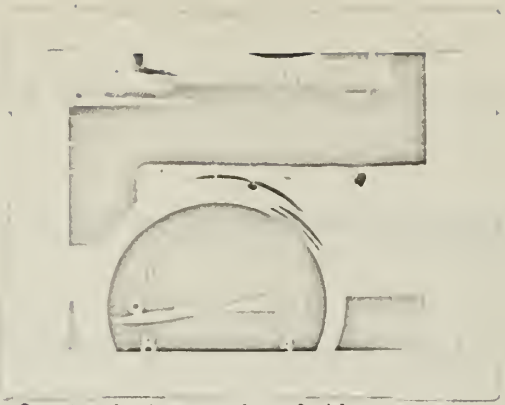
5. Low amplitude but steep internal waves are formed over the sloping shelf.



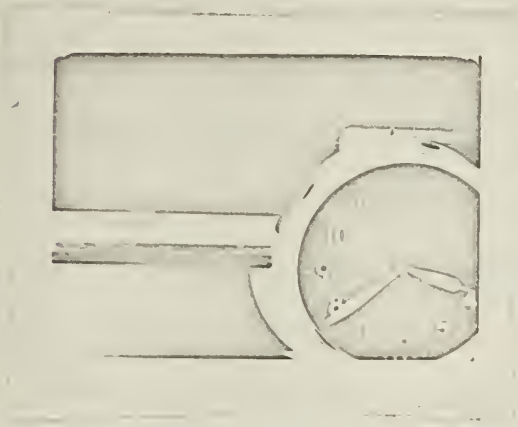
6. A distant view illustrating the dissipation of the internal waves seaward of the shelf.



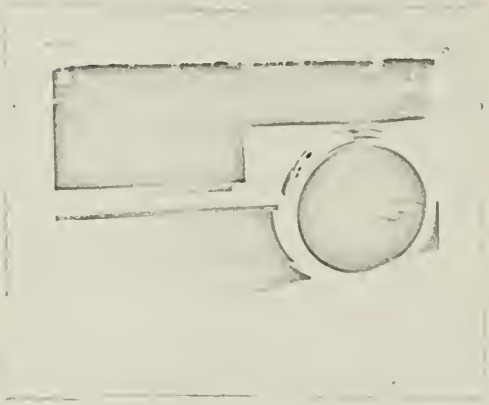
7. Standing internal waves over the shelf when there is little friction.



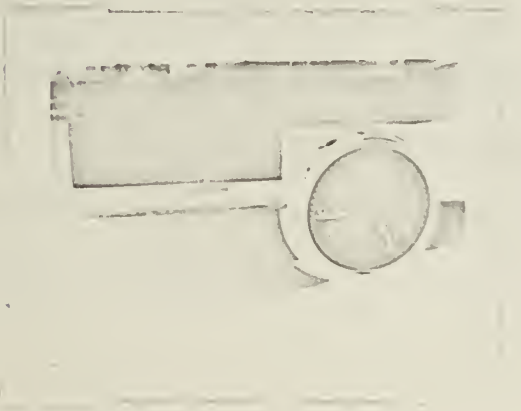
8. A photograph of the same wave shown in 7, taken a quarter cycle later.



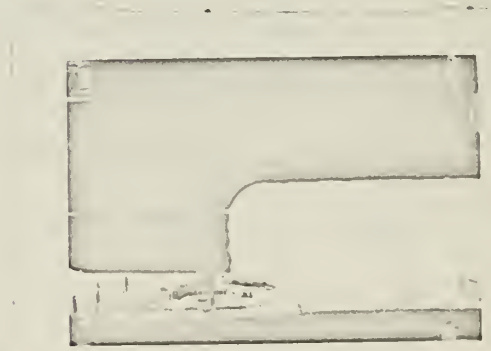
9. A breaking internal wave at the shelf break is shown just to the left of the shelf.



10. Rapid dissipation of the internal wave over the shelf is shown when friction is important.



11. Internal waves formed when the pycnocline is below the shelf.



12. A close-up view over the shelf showing the mixing which has spread the two strips of dye.

BIBLIOGRAPHY

Defant, Albert, 1950: On the origin of internal tide waves in the open sea. J. Mar. Res., 9, pp. 111-119.

Ekman, V . W., 1931: On internal waves. Rapp et Proces-Verbaux des Reunions, LXXVI, pp. 5-31.

Emery, K. O. and H. J. Summers, 1960: Internal waves of tidal period off Southern California. Unpublished manuscript.

Fjeldstad, J. E., 1935: Interne wellen. Geophys. Publ., 10, 6, pp. 1-35.

_____, 1952: Observations of internal waves in gravity waves. Nat. Bur. Stds. Circ. 521, pp. 39-45.

Haurwitz, Bernhard, 1948: The effect of ocean currents on internal waves. J. Mar. Res., 7, pp. 217-228.

_____, 1950: Internal waves of tidal character. Trans. Amer. Geophys. Union, 31, pp. 47-52.

_____, 1952: On the reality of internal lunar tidal waves in the ocean. WHOI Tech. Rep. Ref. 52-71, p. 26.

_____, 1953: Internal waves in the ocean. WHOI Tech. Rep. Ref. 53-69, p. 43.

Helland-Hansen, B., and F. Nansen, 1909: The Norwegian Sea. Report on Norwegian Fish and Marine Investigation. Kristiania (Oslo), pp. 87-132.

LaFond, E. C., 1961: The isotherm follower. J. Mar. Res., 19, 1, pp. 33-39.

Mortimer, C. H., 1952: Study of internal waves in Lake Windermere. Phil. Trans. (B), 230, pp. 355-404.

Rattray, Maurice, Jr., 1957: Propagation and dissipation of long internal waves. Trans. Amer. Geophys. Union, 38, pp. 495-500.

_____, 1960: On the coastal generation of internal tides. Tellus, 12, 1, pp. 54-62.

Rudnick, P., and J. D. Cochrane, 1951: Diurnal fluctuations in bathythermograms. J. Mar. Res., 10, pp. 257-262.

Reid, Joseph L., Jr., 1956: Observations of internal tides in October 1950. Trans. Amer. Geophys. Union, 37, pp. 278-286.

Sandstrom, J., 1908: Dynamische versuche mit meerwasser. Ann. Hydr. Mar. Met., 36, 1, pp. 6-23.

Seiwell, H. R., 1939: The effect of short period variations of temperature and salinity on calculations in dynamic oceanography. Pap. Phys. Ocean. Met., 7, 3, p. 32.

Stokes, G., 1847: On the theory of oscillatory waves. Cambridge Trans., VIII, Part IV, pp. 441-455.

Zeilon, Nils, 1912: On tidal boundary-waves and related hydrodynamical problems. Kungl. Svenska Vetenskapakademiens Handlingar, 47, 4, pp. 1-46.

_____, 1913: On the seiches of the Gullmar Fiord. Svensk. Hydr-Biol. Komm. Skrifter, 5, pp. 1-22.

_____, 1934: Experiments on boundary tides. Goteborg Vetensk Samh. Handl., Folj 5, Ser. B, 3, 10, p. 8.

APPENDIX I

LIST OF SYMBOLS

u	horizontal velocity
h	depth of the upper layer
h'	depth of the lower layer
d	depth over the shelf
D	depth seaward of the shelf
l	width of the shelf
$\Delta\rho$	difference in density between the upper and lower layer
ρ	density of the lower layer
g	acceleration of gravity
ζ	elevation of the upper surface
ζ'	elevation of the pycnocline
σ	angular frequency of the waves
P	dimensionless parameter, $P = \left(\frac{\sigma^2}{g\frac{\Delta\rho}{\rho}h}\right)^{\frac{1}{2}} l$
A	amplitude of the internal wave over the shelf
B	amplitude of the internal wave seaward of the shelf

APPENDIX II

SAMPLE CALCULATIONS

II.1 Development of the Theoretical Curves

The theoretical relationships as shown in Figures 5, 6, and 7 were developed to permit the comparison of the model results with Rattray's theory of internal wave generation. From Section 1.4.3, expressions for the amplitude of the internal waves are

$$B = \rho_0 h \left(\frac{1}{d} - \frac{1}{D}\right) \frac{k_2 l \cos k_1 l - \frac{k_2}{k_1} \sin k_1 l}{\sqrt{\cos^2 k_1 l + \frac{k_2^2}{k_1^2} \sin^2 k_1 l}}$$

and $$A = \rho_0 h \left(\frac{1}{d} - \frac{1}{D}\right) \sqrt{\frac{1 + k_2^2 l^2}{\cos^2 k_1 l + \frac{k_2^2}{k_1^2} \sin^2 k_1 l}} \; .$$

The depth ratio $\frac{d}{D}$, and the ratio of the thickness of the upper layer to the depth over the shelf $\frac{h}{d}$ can be taken as 0.3 and 0.7 respectively to correspond to the values in the figures. If the parameter P is substituted in the expressions for A and B they are made somewhat easier to work with. The expression for B with $\frac{d}{D}$=0.3 and $\frac{h}{d}$=0.7 and with the parameter P substituted is

$$B = \rho_0 \, 0.49 \left(\frac{1.1 P \cos 1.82P - .19 \sin 1.8P}{\sqrt{\cos^2 1.8P + .04 \sin^2 1.82P}} \right) .$$

The above expression can be graphed by the standard techniques of analytic geometry by finding relative maximums, relative minimums, and zero's. The expression for A can be handled in a similar manner.

II.2 Calculations from the Model Data

Results of the model experiment were displayed by plotting the ratio of internal to surface wave amplitudes against the parameter P. Each plot was made for constant values of the depth ratio $\frac{d}{D}$, and of the ratio of the upper layer thickness to the depth over the shelf $\frac{h}{d}$.

A sample calculation for P is shown below for the data from run 49D.

directly measured quantities

$\sigma = 1.57 \text{ sec}^{-1}$

$D = 28.4 \text{ cm}$

$d = 8.6 \text{ cm}$

$l = 87.5 \text{ cm}$

quantities measured from the salinity profile

$h = 6.0 \text{ cm}$

$\Delta\rho = 0.023 \text{ gm}^{1} \text{ cm}^{-3}$

$\rho = 1.023 \text{ gm}^{1} \text{ cm}^{-3}$

The expression for P is

$$P = \left(\frac{\sigma^2}{g \frac{\Delta\rho}{\rho} h}\right)^{\frac{1}{2}} l$$

Substitution of the measured values in the above expression gives a

$$P = \left(\frac{1.57}{980 \quad \frac{.023}{1.023} \quad 6.0}\right)^{1/2} 87.5$$

Measurements of the internal wave amplitude were made at one-half internal wave length seaward of the shelf break for B and at one-half wave length shoreward of the shelf break for A. The surface wave amplitude was measured 1 meter seaward of the shelf break.

Sample measurements for run 49D are shown below.

$$B = 0.8 \text{ cm}$$

$$A = 0.6 \text{ cm}$$

$$S_o = 0.14 \text{ cm}$$

Calculation of the amplitude ratios gives

$$\left|\frac{B}{S_o}\right| = 5.0$$

$$\left|\frac{A}{S_o}\right| = 4.3$$

Tests were made in two different ways to see if the results of the experiment were reproducible. In the first method, which tested the measurement of the internal and surface wave amplitude, measurements were made at different times during the same run, and the agreement was found to be very good, as shown in the table below.

TABLE 1

Run 27	(cm)	B (cm)	$\left\|\frac{B}{S_o}\right\|$	P
	0.17	1.4	8.2	6.6
	0.18	1.4	7.8	6.5
	0.17	1.45	8.5	6.6

The other test was made to see if the same amplitude ratios were obtained for the same values of P when the value of P was obtained by different combinations of the variables. This method also tested the desirability of using P as the independent parameter of the plot representing the results. The great number of parameters involved in the problem and the inability to control all of them exactly made it difficult to get different combinations of variables to give the same value of P.

The table below does show that the results are reproducable within the limits of error of the measurements.

Run	P	$\left\lvert \frac{B}{J_0} \right\rvert$
10A	7.8	4.7
49B	7.8	4.4
23	7.8	4.3

APPENDIX III

EXPERIMENTAL DATA

Run	σ	D	d	h	$\Delta\rho$	P	$2\zeta_o$	2" seaward	l	2" over shelf	λ	shelf type
1	2.25	23.5	4.3	3.4	.015	16.7	.35	2.0	58.5	-	13.0	flat
1A	2.20	"	"	"	"	"	"	1.5	"	-	12.6	"
8C	1.0	29.6	8.1	5.0	.027	5.35	.35	1.0	"	-	17.3	"
8D	"	"	"	"	"	"	"	.95	"	-	"	"
8E	"	"	"	"	"	"	"	1.1	"	-	19.1	"
9A	1.35	29.3	7.8	5.0	.028	6.5	.41	1.2	"	-	19.0	"
9A2	1.35	"	"	"	"	"	"	1.4	"	-	18.8	"
9C	1.46	"	"	"	.027	7.45	.43	1.2	"	-	11.1	"
9C2	"	"	"	"	"	"	"	1.2	"	-	"	"
10A	1.19	28.4	6.9	3.5	.023	7.80	.36	1.7	"	-	17.1	"
10B1	0.95	"	"	"	.022	7.07	.30	1.1	"	-	19.8	"
10B2	1.90	"	"	"	"	13.6	.44	0.9	"	-	13.2	"
10B3	1.28	"	"	"	"	8.62	.36	1.6	"	-	14.7	"
10C	1.18	"	"	"	"	7.80	"	2.1	"	-	16.9	"
13	1.75	28.7	7.2	3.4	.018	1.47	.41	~0	6.5	-	-	"

(continued)

Appendix III (continued)

Run	σ	D	d	h	$\Delta\rho$	P	$2f_o$	$2f'$ seaward	l	$2f'$ over shelf	λ	shelf type
13B	0.82	28.7	7.2	3.4	.018	0.32	.23	~0	6.5	-	-	flat
15A3	1.23	29.2	7.7	3.3	.020	2.74	.39	0.9	15.5	-	10.1	"
15B	"	"	"	"	"	"	.39	0.6	"	-	14.1	"
17	1.21	28.2	6.7	4.0	.028	1.78	.34	0.9	"	-	23.7	"
19	1.39	"	"	"	.027	5.82	.38	1.2	43.0	-	12.0	"
20	1.39	"	"	"	"	"	"	1.3	"	-	11.0	"
21	1.45	"	"	"	"	6.22	"	1.6	"	-	8.8	"
22	1.21	25.1	2.5	4.0	.022	5.60	.30	2.2	"	-	16.1	"
25	1.16	"	"	3.7	.024	8.30	.29	2.4	66.5	-	13.5	"
25B	0.96	"	"	"	"	6.77	.24	2.5	"	-	18.5	"
26	1.50	27.9	6.5	5.0	.021	7.93	.38	1.4	53.5	0.9	13.1	"
27	1.21	"	"	"	"	6.35	.34	2.9	"	1.4	10.2	"
28	1.16	27.5	6.1	4.5	.021	9.37	.31	1.8	77.5	0.8	11.8	"
29	1.03	27.9	-	4.5	.020	9.45	.30	2.9	"	-	25.0	slope
31	1.40	27.4	-	4.3	"	9.07	.32	2.9	"	-	15.9	"
32A2	1.14	26.7	-	4.4	"	9.53	.28	1.2	"	-	8.4	"

(continued)

Appendix III (continued)

Run	σ	D	d	h	$\Delta\rho$	P	$2f_0$	$2f'$ seaward	l	$2f'$ over shelf	λ	shelf type
32A3	0.92	26.7	-	4.4	.020	7.68	.25	1.1	77.5	-	15.8	slope
32B	"	"	-	"	"	5.05	"	1.0	51.0	-	14.6	"
34	1.25	26.0	-	"	"	6.85	.22	0.6	"	-	12.5	"
34B1	1.03	"	-	"	"	4.65	.27	1.0	"	-	16.9	"
34B2	1.39	26.0	-	"	"	8.50	.35	0.8	"	0.6	8.8	"
35A2	1.25	28.7	-	5.0	"	6.55	.37	2.2	51.5	0.7	16.9	"
36B	1.16	"	6.8	5.2	.018	6.25	.35	1.6	"	1.2	14.5	"
36B2	1.02	"	"	"	"	5.60	.32	1.2	"	-	22.5	"
40	0.88	31.2	9.3	6.0	.016	4.98	.36	0.6	"	-	12.8	flat
41B	0.98	"	"	"	"	5.25	.35	2.3	"	-	6.4	"
42B	1.40	29.4	-	5.1	.033	6.32	.27	1.1	58.4	-	25.3	slope
44	1.46	"	-	5.0	.031	6.92	.24	0.9	"	-	17.3	"
45	1.80	28.8	8.8	5.2	.032	8.05	.23	0.8	57.4	0.7	11.2	flat
47	1.75	26.2	6.3	4.8	.031	8.34	.18	1.2	57.5	-	14.0	"
47B	"	"	"	"	"	"	.28	1.6	"	0.9	15.2	"
48	1.77	23.2	3.2	5.5	.030	8.00	.14	2.3	"	-	16.4	"

(continued)

Appendix III (continued)

Run	σ	D	d	h	$\Delta\rho$	P	$2\mathcal{F}_0$	$2\mathcal{F}'$ seaward	l	$2\mathcal{F}'$ over shelf	λ	shelf type
48B	1.26	23.2	3.2	5.5	.030	5.70	.13	2.0	57.5	-	15.3	flat
48C	0.93	"	"	"	"	4.20	.10	1.3	"	-	11.8	"
49A	1.96	28.4	8.6	6.0	.023	9.70	.30	0.6	"	-	9.2	"
49B	1.57	"	"	"	"	7.77	.27	1.2	"	0.9	10.8	"
49C	"	"	"	"	"	4.77	.27	0.9	35.3	0.9	9.6	"
49D	"	"	"	"	"	11.83	"	1.6	87.5	1.2	10.1	"
49E	1.19	"	"	"	"	8.96	.24	2.1	"	-	22.2	"

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